corresponds to the mth mode; thus, in principle, c_m may be arbitrarily large, although admittedly the corresponding distributions of lift may not be of any practical interest.

Finally, it is necessary to call attention to the fact that Barrows' quotation of reciprocity theorem for wing rolling moment for an arbitrary imposed angle-of-attack distribution is not generally true within the framework of lifting-surface theory. To paraphrase his statement in corrected form, "The rolling moment on a wing encountering an arbitrary downwash field is equal to the integral over the span of the product of the local angle of attack and the sectional lift at the corresponding spanwise station of a flat-plate wing of identical planform *in reverse flow* which is rolling at a rate $p=2U_0/b$." (The added condition is italicized.)

The general reciprocity relations in lifting-surface theory are between wings in reverse flow. However, as pointed out in Ref. 8, the requirement for flow reversal does not apply in some cases, e.g., within the context of lifting-line theory or slender-wing theory or for wings that are symmetric fore and aft about a spanwise axis of symmetry.

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Reply by Author to A.H. Flax

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I AM indebted to Flax for pointing out the original derivation of the simple rolling-moment relation. As might be surmised from my paper, I had a feeling this result might have been derived earlier. It certainly is a disappointment to have missed an opportunity to quote the immortal Max Munk.

The results quoted for rolling moments were not actually derived by Victory. She merely reproduced an appendix from an earlier report by Hirst and Brooke.[1] They calculated the first four even terms A_n of the Prandtl-Glauert series using the method of least squares with eight control points along the span – a lot of work in those days. Referring to Eq. (31) of my paper, we see that my approximation of c is $c=4(1+\epsilon)$ for the case of $\alpha=y$ (linear antisymmetric angle of attack) which comes reasonably close to their results. I am sure Flax agrees that the proper way to get around the fact that c varies whenever the angle of attack is other than linear antisymmetric is to use a reciprocal relation.

One can only guess at what motivation Flax has for bringing up the 1948 work of Sears. This work originated from a desire to solve the lifting-line equation along the lines of more classical solutions to integral equations. It did result in avoiding the need to solve simultaneous equations for each new angle-of-attack distribution. However, the price paid was too high. A series of eigenvalues and eigenfunctions must be derived for each new wing planform, and these replace the universal sines and cosines used by Glauert. The later introduction of the reciprocity theorems provided a much cleaner and simpler means of obtaining similar results. The reciprocity theorems require a single spanwise integration, whereas the Sears solution requires a whole series of integrations (except in the unusual situation mentioned by Flax).

Flax is quite correct in pointing out that to maintain generality one must include the words "in reverse flow" in the reciprocal theorem. As he so kindly points out, within the context of lifting line theory, there is no requirement for flow reversal, so the results derived in the paper are unaffected. However, this does raise the following questions: When is flow reversal required? Is it required for swept wings? The answer to the latter is "probably not" for purposes of calculating vortex encounter motions.

The upshot of my analysis is that by calculating the rolling moment in the manner prescribed by Eggleston and Diederich (Ref. 9 of my paper), the main effect of taper is contained in the roll-damping derivative; and there are only secondary effects caused by a spanwise redistribution of the loading. The same is probably true of the effects of sweep. Another reciprocal theorem of Heaslet and Spreiter (Ref. 8 of my paper), which carries no restrictions on planform, appears as follows:

> "The rolling moment per unit angular rolling velocity of flat-plate wings in steady or indicial motion is the same in forward and reverse flight."

The same statement cannot be made about pitching moments, incidentially. Using this additional theorem, we see that to first order (that is, to within the accuracy of the first term of my Eq. 38) flow reversal has no effect. Three conditions must prevail simultaneously before there is a practical effect of including flow reversal in the calculation:

1) The following wing must be substantially swept.

2) The oncoming downwash field must differ significantly from solid body rotation.

3) Predictions of rolling moment are required that are more accurate than those available from the first-order solution.

The last condition above seems very unlikely in view of the uncertainties of predicting the dynamics of a vortex encounter.

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